Supergravity algebras and Minkowski vacua in $\mathcal{N} = 1$ generalised flux compactifications

Adolfo Guarino

Instituto de Física Teórica UAM-CSIC, Madrid

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Work in collaboration with B. de Carlos and J. M. Moreno.

Motivation & objectives

- Why generalised fluxes ?
  - To restore duality symmetries at the 4D SUGRA models level.
  - Flux induced $V(\Phi)$ for all IIA/IIB closed string moduli: moduli stabilisation at dS vacua, SUSY breaking, modular inflation...
- What do we want to present in this talk ?
  - A complete set of consistent T-duality invariant $\mathcal{N} = 1$ SUGRA models which are interesting for phenomenology.
  - The interplay between the flux induced Supergravity algebras and the structure of classical Minkowski extrema for the moduli fields.

Outlook

Fluxes, symmetries and Supergravity algebras

Objective

Classify the isotropic flux-induced Supergravity algebras underlying the set of $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ type II orientifold models.

Key points

- Make an appropriate *duality frame* choice.
- Use the $\mathbb{Z}_2 \times \mathbb{Z}_2$ isotropic orbifold symmetry.

Generalised fluxes, T-duality and 12d algebra

- T-duality transformations give rise to generalised NS-NS flux backgrounds

$$\bar{H}_{abc} \xrightarrow{T_a} \omega_{bc}^{a} \xrightarrow{T_b} Q_c^{ab} \xrightarrow{T_c} R^{abc}$$

including *(geo)metric*, ω, and *non-geometric* Q and R fluxes.

- Proposal for a T-duality invariant 12d algebra, $\mathfrak{g}$, spanned by X^a (gauge) and Z_a (isometry) generators, with $a = 1, ..., 6$,

$$\begin{array}{rcrcl} [Z_a\,,Z_b] & = & \bar{H}_{abc}\,X^c & + & \omega_{ab}^{c}\,Z_c \\ [Z_a\,,X^b] & = & -\,\omega_{ac}^{b}\,X^c & + & Q_a^{bc}\,Z_c \\ [X^a,X^b] & = & Q_c^{ab}\,X^c & + & R^{abc}\,Z_c \end{array}$$

Shelton, Taylor and Wecht [arXiv:hep-th/0508133]

totally induced by the generalised NS-NS flux sector playing the role of structure constants $\Rightarrow$ Jacobi constraints.

$\mathcal{N} = 1$ type II orientifold limits on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

- $\mathcal{N} = 1$ low energy effective theories based on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold are promising for moduli stabilisation at classical dS extrema.

  Caviezel, Koerber, Körs, Lüst, Tsimpis and Zagermann [arXiv:hep-th/08063458]
  Flauger, Paban, Robbins and Wrase [arXiv:hep-th/08123886]
  Caviezel, Koerber, Körs, Lüst, Wrase and Zagermann [arXiv:hep-th/08123551]

- Type II supergravities on $\frac{\mathbb{T}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$ orbifold $\Rightarrow \mathcal{N} = 2$ Supergravities further broken to $\mathcal{N} = 1$ in the orientifold limits.

- The orientifold projections allow for Op-planes and project half of the flux entries out of the theory $\Rightarrow$ (T-) duality frames.

Type IIB with O3/O7 planes: Fluxes and algebra

- If working in type IIB with O3/O7-planes $\Rightarrow$ Only $\bar{H}_3$ and Q fluxes.

- This duality frame is suitable for classifying the algebras

$$
\begin{aligned}
[Z_a\,, Z_b] &= \bar{H}_{abc}\, X^c &\rightarrow&\quad \text{Extension from } \mathfrak{g}_{gauge} \text{ to } \mathfrak{g}\,.\\
[Z_a\,, X^b] &= Q_a^{bc}\, Z_c &\rightarrow&\quad \text{Co-adjoint action } Q^* \text{ of } Q\,.\\
[X^a, X^b] &= Q_c^{ab}\, X^c &\rightarrow&\quad \mathfrak{g}_{gauge}\ Q\text{-subalgebra}\,.
\end{aligned}
$$

- In the double space formalism, this corresponds to having a T-fold space: the X^a vectors generate $\mathcal{G}_{gauge}$, while the Z_a vectors generate the coset space $\mathcal{G}/\mathcal{G}_{gauge}$.

- Quadratic Jacobi identities $\Rightarrow$ $Q^2 = 0$ and $\bar{H}_3\, Q = 0$.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ isotropic orbifold and algebra spliting

- General arguments can be used to determine the set of allowed $\mathfrak{g}$ algebras in the type IIB with O3/O7 duality frame.

- $(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold symmetry + $\mathbb{Z}_3$ isotropy on the fluxes $\Rightarrow$ $\mathfrak{g}$ can be classified according to the group $SO(2,2) \times SO(3) \subset SO(6,6)$ with the embedding $(\mathbf{4},\mathbf{3}) = \mathbf{12} \Rightarrow \mathfrak{g}$ splits into four subspaces endowed with a ϵ_{IJK} cyclic structure.

  Derendinger, Kounnas, Petropoulos and Zwirner [arXiv:hep-th/0411276]

- This makes the simple $\mathfrak{so}(3) \sim \mathfrak{su}(2)$ algebra to be the fundamental block $\Rightarrow$ $\mathfrak{g}_{gauge}$ comes from gluing together two $\mathfrak{su}(2)$ factors.

- The set of $\mathfrak{g}_{gauge}$: $\mathfrak{so}(3,1)$, $\mathfrak{so}(4)$, $\mathfrak{iso}(3)$, $\mathfrak{nil}$ and $\mathfrak{su}(2) + \mathfrak{u}(1)^3$.

Font, A.G, and Moreno [arXiv:0809.3748 [hep-th]]

The set of compatible B-field reductions

- Denoting $(E^I, \widetilde{E}^I)_{I=1,2,3}$ a basis for $\mathfrak{g}_{gauge}$, the entire set of subalgebras is gathered in the gauge brackets

$$[E^I, E^J] = \kappa_1 \, \epsilon_{IJK} \, E^K \quad , \quad [E^I, \widetilde{E}^J] = \kappa_{12} \, \epsilon_{IJK} \, \widetilde{E}^K \quad , \quad [\widetilde{E}^I, \widetilde{E}^J] = \kappa_2 \, \epsilon_{IJK} \, E^K$$

restricted by Jacobi to the branches

$$\kappa_1 = \kappa_{12} \qquad \text{or} \qquad \kappa_{12} = \kappa_2 = 0$$

- We will refer to these brackets as the canonical form of $\mathfrak{g}_{gauge}$.

- Five non-equivalent $\mathfrak{g}_{gauge} \Rightarrow$ **Five non-equivalent B-field reductions**.

$$\mathfrak{so}(3,1) \quad , \quad \mathfrak{so}(3) \quad , \quad \mathfrak{iso}(3) \quad , \quad \mathfrak{nil} \quad , \quad \mathfrak{su}(2) + \mathfrak{u}(1)^3$$

The extension to a full Supergravity algebra

- Question: How to go from $\mathfrak{g}_{gauge}$ to a full 12d algebra $\mathfrak{g}$?

- In addition to the gauge brackets, the algebra $\mathfrak{g}$ will also involve a new set of isometry generators denoted $(D_I, \widetilde{D}_I)_{I=1,2,3}$.

- The mixed gauge-isometry brackets are demanded to be given by the co-adjoint action of the $\mathfrak{g}_{gauge}$ structure constants $\Rightarrow$ Not extra parameters added.

- The isometry-isometry brackets are only demanded to satisfy the 12d Jacobi identities $\Rightarrow$ Adding two real (ϵ_1, ϵ_2) degrees of freedom determining the extension from $\mathfrak{g}_{gauge}$ to $\mathfrak{g}$.

The extensions

- The most general 12d brackets are given by

$\kappa_1 = \kappa_{12}$	E^J	$\widetilde{E}^J$	D_J	$\widetilde{D}_J$
E^I	$\kappa_1\, E^K$	$\kappa_1\, \widetilde{E}^K$	$\kappa_1\, D_K$	$\kappa_1\, \widetilde{D}_K$
$\widetilde{E}^I$	$\kappa_1\, \widetilde{E}^K$	$\kappa_2\, E^K$	$-\kappa_2\, \widetilde{D}_K$	$-\kappa_1\, D_K$
D_I	$\kappa_1\, D_K$	$-\kappa_2\, \widetilde{D}_K$	$-\epsilon_1\,\kappa_2\, E^K - \epsilon_2\,\kappa_2\, \widetilde{E}^K$	$\epsilon_2\,\kappa_2\, E^K + \epsilon_1\,\kappa_1\, \widetilde{E}^K$
$\widetilde{D}_I$	$\kappa_1\, \widetilde{D}_K$	$-\kappa_1\, D_K$	$\epsilon_2\,\kappa_2\, E^K + \epsilon_1\,\kappa_1\, \widetilde{E}^K$	$-\epsilon_1\,\kappa_1\, E^K - \epsilon_2\,\kappa_1\, \widetilde{E}^K$

$\kappa_{12} = \kappa_2 = 0$	E^J	$\widetilde{E}^J$	D_J	$\widetilde{D}_J$
E^I	$\kappa_1\, E^K$	0	$\kappa_1\, D_K$	0
$\widetilde{E}^I$	0	0	0	0
D_I	$\kappa_1\, D_K$	0	$-\epsilon_1\,\kappa_1\, E^K$	0
$\widetilde{D}_I$	0	0	0	$-\epsilon_2\,\kappa_1\, \widetilde{E}^K$

- We will also refer to them as the canonical form of $\mathfrak{g}$ $\Rightarrow$ This form involves the parameters $\kappa_{1,2}$ and $\epsilon_{1,2}$.

Semisimple B-field reductions

$\mathfrak{g}_{gauge}$	$\mathfrak{g}$	EXTENSION
$\mathfrak{so}(3,1)$	$\mathfrak{so}(3,1)^2$	$\epsilon_1^2 + \epsilon_2^2 \neq 0$
	$\mathfrak{so}(3,1) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$\epsilon_1^2 + \epsilon_2^2 = 0$
$\mathfrak{so}(4)$	$\mathfrak{so}(3,1)^2$	$(\epsilon_1 + \epsilon_2) > 0 \quad , \quad (\epsilon_1 - \epsilon_2) > 0$
	$\mathfrak{iso}(3)^2$	$(\epsilon_1 + \epsilon_2) = 0 \quad , \quad (\epsilon_1 - \epsilon_2) = 0$
	$\mathfrak{so}(4)^2$	$(\epsilon_1 + \epsilon_2) < 0 \quad , \quad (\epsilon_1 - \epsilon_2) < 0$
	$\mathfrak{so}(3,1) + \mathfrak{iso}(3)$	$(\epsilon_1 + \epsilon_2) \geqslant 0 \quad , \quad (\epsilon_1 - \epsilon_2) \geqslant 0$
	$\mathfrak{so}(3,1) + \mathfrak{so}(4)$	$(\epsilon_1 + \epsilon_2) \gtrless 0 \quad , \quad (\epsilon_1 - \epsilon_2) \lessgtr 0$
	$\mathfrak{iso}(3) + \mathfrak{so}(4)$	$(\epsilon_1 + \epsilon_2) \leqslant 0 \quad , \quad (\epsilon_1 - \epsilon_2) \leqslant 0$

de Carlos, A.G, and Moreno [arXiv:0907.5580 [hep-th]]

Non-semisimple B-field reductions

$\mathfrak{g}_{gauge}$	$\mathfrak{g}$	EXTENSION	
$\mathfrak{iso}(3)$	$\mathfrak{so}(3,1) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$\epsilon_1 > 0$	$\epsilon_2 = free$
	$\mathfrak{iso}(3) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$\epsilon_1 = 0$	
	$\mathfrak{so}(4) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$\epsilon_1 < 0$	
$\mathfrak{nil}$	$\mathfrak{nil}_{12}(4)$	$\epsilon_1 = free$	$\epsilon_2 \neq 0$
	$\mathfrak{nil}_{12}(2)$		$\epsilon_2 = 0$
$\mathfrak{su}(2) + \mathfrak{u}(1)^3$	$\mathfrak{so}(3,1) + \mathfrak{nil}$	$\epsilon_1 > 0$	$\epsilon_2 \neq 0$
	$\mathfrak{so}(3,1) + \mathfrak{u}(1)^6$		$\epsilon_2 = 0$
	$\mathfrak{iso}(3) + \mathfrak{nil}$	$\epsilon_1 = 0$	$\epsilon_2 \neq 0$
	$\mathfrak{iso}(3) + \mathfrak{u}(1)^6$		$\epsilon_2 = 0$
	$\mathfrak{so}(4) + \mathfrak{nil}$	$\epsilon_1 < 0$	$\epsilon_2 \neq 0$
	$\mathfrak{so}(4) + \mathfrak{u}(1)^6$		$\epsilon_2 = 0$
$\mathfrak{u}(1)^6$	$\mathfrak{nil}_{12}(2)$	UNCONSTRAINED	

de Carlos, A.G, and Moreno [arXiv:0907.5580 [hep-th]]

- If $\mathfrak{g}_{gauge} = \mathfrak{u}(1)^6 \Rightarrow$ only gauge fluxes, i.e. $\mathfrak{g} = \mathfrak{nil}^2$.

  Roest [arXiv:0902.0479 [hep-th]]

Review

- If we are given a background for the Q and $\bar{H}_3$ fluxes satisfying $Q^2 = 0$ and $\bar{H}_3 Q = 0$, then it will correspond to one of the previously discussed algebras with a non-canonical embedding within the fluxes.
- Provided a B-field reduction based on a $\mathfrak{g}_{gauge}$, its extension to a full 12d algebra $\mathfrak{g}$ is totally in the two (ϵ_1, ϵ_2) real parameters.

Question

- At the SUGRA models level, which are the consequences of bringing the brackets induced by the Q and $\bar{H}_3$ fluxes into their canonical form?

Flux induced Supergravity models

Objective

Following with the $\mathbb{Z}_2 \times \mathbb{Z}_2$ isotropic orbifold, we want to derive the characteristic $\mathcal{N} = 1$ flux-induced SUGRA models for the five non-equivalent B-field reductions previously found.

Key points

- The choice of the embedding of $\mathfrak{g}$ within the Q and $\bar{H}_3$ fluxes becomes a symmetry of the SUGRA models up to a global volume factor.
- The R-R flux sector can be efficiently parameterised making use of the axion shift symmetries.

Type IIB with O3/O7: Fluxes and effective action

- The set of isotropic flux backgrounds comprises the $\bar{H}_3$ and Q fluxes in the generalised NS-NS sector together with a $\bar{F}_3$ flux in the R-R sector.
- The Ansatz of isotropic fluxes is compatible with vacua in which the geometric moduli are also isotropic $\Rightarrow$ One complex structure modulus U + one Kähler modulus T + the axio-dilaton S.
- The $\mathcal{N}=1$ effective action is defined by

$$\begin{aligned} K &= -3\,\log\left(-i\,(U-\bar{U})\right) - \log\left(-i\,(S-\bar{S})\right) - 3\,\log\left(-i\,(T-\bar{T})\right) \\ W &= \int_Y \left(\bar{F}_3 \wedge \Omega\right) - S\left(\bar{H}_3 \wedge \Omega\right) + \left(Q\,\mathcal{J} \wedge \Omega\right) \end{aligned}$$

Aldazabal, Cámara, Font and Ibáñez [arXiv:hep-th/0602089]

where $\Omega(U)$ is the holomorphic 3-form and $\mathcal{J}(T)$ is the complexified Kähler 4-form.

- Computing the superpotential the superpotential,

$$W(U,S,T) = \underbrace{P_1(U)}_{\bar{F}_3} + \underbrace{P_2(U)}_{\bar{H}_3} S + \underbrace{P_3(U)}_{Q} T$$

it involves cubic polynomials in the complex structure modulus U.

- The coefficients in $P_2(U)$ and $P_3(U)$ expand the $\bar{H}_3$ and Q fluxes respectively $\Rightarrow$ restricted by Jacobi identities.

- Only Supersymmetric vacua structure found.

Shelton, Taylor and Wecht [arXiv:hep-th/0607015]
Font, A.G, and Moreno [arXiv:0809.3748 [hep-th]]

A new approach: The characteristic $\mathfrak{g}_{gauge}$ based SUGRA models

- Provided a consistent background for the Q and $\bar{H}_3$ fluxes in a type IIB with O3/O7 orientifold model, it can always be taken into the canonical form of $\mathfrak{g}$ by applying a rotation of the form

$$\begin{pmatrix} E^I \\ \widetilde{E}^I \end{pmatrix} = \frac{\Gamma}{|\Gamma|^2} \begin{pmatrix} -X^{2I-1} \\ X^{2I} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -D_I \\ \widetilde{D}_I \end{pmatrix} = \frac{\text{Adj}(\Gamma)}{|\Gamma|^2} \begin{pmatrix} -Z_{2I-1} \\ Z_{2I} \end{pmatrix}$$

via the general $\Gamma \in \text{GL}(2,\mathbb{R})$ matrix, $\Gamma \equiv \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$.

- After performing this rotation: $Q = Q(\kappa_i)$ and $\bar{H}_3 = \bar{H}_3(\kappa_i, \epsilon_i)$

- The κ_1 and κ_2 parameters in the gauge brackets can be normalised to +1, 0, -1 via a rescaling of the $\left\{E^I, \widetilde{E}^I\right\}$ generators $\Rightarrow$ rescaling of Γ.

- At the effective SUGRA models level, this rotation translates into a transformation on the U modulus

$$U \rightarrow \mathcal{Z} \equiv \Gamma U = \frac{\alpha U + \beta}{\gamma U + \delta}$$

and the generalised NS-NS flux induced polynomials result in

$$P_2(U) = (\gamma U + \delta)^3 \mathcal{P}_2(\mathcal{Z}) \quad , \quad P_3(U) = (\gamma U + \delta)^3 \mathcal{P}_3(\mathcal{Z})$$

	$\mathcal{P}_3(\mathcal{Z})/3$	$\mathcal{P}_2(\mathcal{Z})$
$\kappa_1 = \kappa_{12}$	$\kappa_2 \mathcal{Z}^3 - \kappa_1 \mathcal{Z}$	$\kappa_2 \left(\epsilon_1 \mathcal{Z}^3 + 3\,\epsilon_2 \mathcal{Z}^2\right) + \kappa_1 \left(\epsilon_2 + 3\,\epsilon_1 \mathcal{Z}\right)$
$\kappa_{12} = \kappa_2 = 0$	$\kappa_1 \mathcal{Z}$	$\kappa_1 \left(\epsilon_1 \mathcal{Z}^3 + \epsilon_2\right)$

- The polynomial $\mathcal{P}_3(\mathcal{Z})$ results totally fixed after the B-field reduction choice while $\mathcal{P}_2(\mathcal{Z})$ depends on the $\epsilon_{1,2}$ parameters specifying its extension to a full 12d algebra.

- In terms of the $\mathcal{Z}$ modulus, the R-R $\bar{F}_3$ flux-induced polynomial

$$P_1(U) = (\gamma U + \delta)^3 \mathcal{P}_1(\mathcal{Z})$$

can be conveniently expanded as

$$\mathcal{P}_1(\mathcal{Z}) = \xi_s \, \mathcal{P}_2(\mathcal{Z}) \, + \, \xi_t \, \mathcal{P}_3(\mathcal{Z}) \, - \, \xi_3 \widetilde{\mathcal{P}}_2(\mathcal{Z}) \, + \, \xi_7 \, \widetilde{\mathcal{P}}_3(\mathcal{Z})$$

where $\widetilde{\mathcal{P}}_i(\mathcal{Z})$ denotes the dual of $\mathcal{P}_i(\mathcal{Z})$ such that $\mathcal{P}_i \rightarrow \frac{\widetilde{\mathcal{P}}_i}{\mathcal{Z}^3}$ when $\mathcal{Z} \rightarrow -\frac{1}{\mathcal{Z}}$.

- This parametrization allows us to remove the R-R flux degrees of freedom, (ξ_s, ξ_t) , from the effective theory through the real shifts

$$\mathcal{S} = S + \xi_s \qquad , \qquad \mathcal{T} = T + \xi_t$$

on the dilaton and the Kähler moduli fields $\Rightarrow$ This leaves us with two (ξ_3, ξ_7) real parameters which relate to localised sources.

- The modulus redefinition $U \to \mathcal{Z}$ corresponds to a Kähler transformation $e^K|W|^2 \to e^{\mathcal{K}}|\mathcal{W}|^2$ of the model to an equivalent one described by

$$\mathcal{K} = -3\,\log\left(-i\,(\mathcal{Z}-\bar{\mathcal{Z}})\right) - \log\left(-i\,(\mathcal{S}-\bar{\mathcal{S}})\right) - 3\,\log\left(-i\,(\mathcal{T}-\bar{\mathcal{T}})\right)$$

$$\mathcal{W} = |\Gamma|^{3/2}\left[\mathcal{T}\,\mathcal{P}_3(\mathcal{Z}) + \mathcal{S}\,\mathcal{P}_2(\mathcal{Z}) - \xi_3\widetilde{\mathcal{P}}_2(\mathcal{Z}) + \xi_7\,\widetilde{\mathcal{P}}_3(\mathcal{Z})\right]$$

Font, A.G, and Moreno [arXiv:0809.3748 [hep-th]]

Review

Provided a B-field reduction based on a $\mathfrak{g}_{gauge}$ gauge subalgebra, the resulting SUGRA models are totally determined by two NS-NS like (ϵ_1, ϵ_2) parameters plus two R-R like (ξ_3, ξ_7) parameters.

The Minkowski solutions

Objective

To find the **complete** set of Minkowski moduli extrema for the set of $\mathcal{N} = 1$ SUGRA models based on the five non-equivent B-field reductions.

Key points

- The stabilisation of the $\mathcal{S}$ and $\mathcal{T}$ moduli field in a Mkw vacuum can be analytically computed.
- After using the scaling properties of the SUGRA models, the parameter space is further reduce to a 2-torus with coordinates $(\theta_\epsilon, \theta_\xi)$.

The equations of motion

- The dynamics of the moduli fields $\Phi \equiv (\mathcal{Z}, \mathcal{S}, \mathcal{T})$ is determined by the standard $\mathcal{N} = 1$ scalar potential

$$V = e^{\mathcal{K}} \left(\sum_{\Phi} \mathcal{K}^{\Phi\bar{\Phi}} |D_{\Phi}\mathcal{W}|^2 - 3|\mathcal{W}|^2 \right)$$

- Moduli fields are stabilised at the minima of the potential taking a vacuum expectation value Φ_0 (VEV) determined by the conditions

$$\left.\frac{\partial V}{\partial \Phi}\right|_{\Phi=\Phi_0} = 0$$

- Our strategy will consist in finding the Mkw extrema, and then to look for dS vacua continuously connected to them via a parameter deformation.

Stabilising the $\mathcal{S}$ and $\mathcal{T}$ moduli

- Since the moduli $\mathcal{S}$ and $\mathcal{T}$ enter the superpotential linearly,

$$e^{-\mathcal{K}}V = \left(1, \mathrm{Re}\mathcal{S}, \mathrm{Re}\mathcal{T}\right) M^{(axi)} \begin{pmatrix} 1 \\ \mathrm{Re}\mathcal{S} \\ \mathrm{Re}\mathcal{T} \end{pmatrix} + \left(1\,,\ \mathrm{Im}\mathcal{S}\,,\ \mathrm{Im}\mathcal{T}\right) M^{(vol)} \begin{pmatrix} 1 \\ \mathrm{Im}\mathcal{S} \\ \mathrm{Im}\mathcal{T} \end{pmatrix}$$

  with $M^{(axi)}$ and $M^{(vol)}$ being 3×3 symmetric matrices that depend on the $\mathcal{Z}$ modulus and on the $\epsilon_{1,2}$ and $\xi_{3,7}$ real parameters.

- At a Mkw extremum, $\dfrac{\partial V}{\partial \mathrm{Im}\Phi} = 0 \Rightarrow \dfrac{\partial(e^{-\mathcal{K}}\,V)}{\partial \mathrm{Im}\Phi} = 0$, and the stabilisation of the moduli $\mathcal{S}$ and $\mathcal{T}$ can be worked out analytically.

Stabilizing the $\mathcal{Z}$ modulus

- Once the $\mathcal{S}$ and $\mathcal{T}$ moduli field stabilisation has been studied analytically, the next step is to study the stabilisation of $\mathcal{Z}$

$$\left.\frac{\partial V}{\mathrm{Re}\mathcal{Z}}\right|_{\Phi=\Phi_0} = \left.\frac{\partial V}{\mathrm{Im}\mathcal{Z}}\right|_{\Phi=\Phi_0} = 0$$

together with the Minkowski condition

$$\left.e^{-\mathcal{K}}V\right|_{\Phi=\Phi_0} = 0$$

- After substituting the $\mathcal{S}_0$ and $\mathcal{T}_0$ VEVs, the system results in a high degree polynomial conditions on the $\mathcal{Z}_0$ modulus components $\Rightarrow$ It has to be takled numerically.

- The SUGRA models based on the $\mathfrak{nil}$ B-field reduction are excluded to accommodate for Mkw extrema $\Rightarrow$ Algebraic geometry techniques.

Scaling properties and parameter space

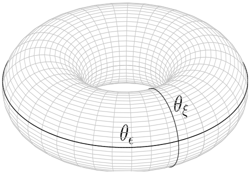
- Applying the parameter redefinitions of

$$\epsilon_1 \to |\epsilon| \cos(\theta_\epsilon) \quad , \quad \epsilon_2 \to |\epsilon| \sin(\theta_\epsilon) \quad , \quad \xi_3 \to \frac{|\xi|}{|\epsilon|} \cos(\theta_\xi) \quad , \quad \xi_7 \to |\xi| \sin(\theta_\xi)$$

and the moduli rescalings $\mathcal{S} \to \frac{\mathcal{S}\,|\xi|}{|\epsilon|}$ and $\mathcal{T} \to \mathcal{T}\,|\xi|$,

$$\mathcal{W} \to |\Gamma|^{\frac{3}{2}}\, |\xi| \;\; \mathcal{W}\,(\Phi\,;\,\theta_\epsilon, \theta_\xi) \qquad \text{and} \qquad V \to \frac{|\Gamma|^3\,|\epsilon|}{|\xi|^2}\; V\,(\Phi\,;\,\theta_\epsilon, \theta_\xi) \;\; .$$

- **The effective parameter space then acquires the topology of a 2-torus with coordinates $(\theta_\epsilon, \theta_\xi)$.**



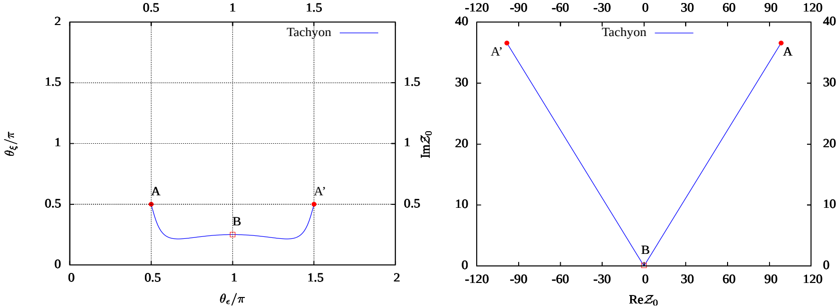
Generalities of the Mkw solutions

- $\text{Im}\Phi_0 > 0$ at any physical Mkw extremum. The distribution of such extrema depends crucially on the sort of SUGRA model it belongs to.

- Non-semisimple B-field reductions $\Rightarrow$ unstable extrema.
  - Only one Mkw extremum $\mathcal{Z}_0^{\star}$ which is rescaled to generate the entire set of them (this can be seen analytically).
  - They draw open lines in both the parameter space $(\theta_\epsilon, \theta_\xi)$ and the complex plane $\mathcal{Z}_0$.

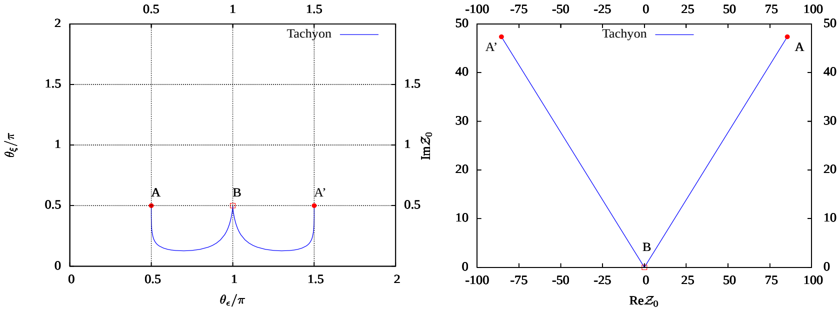
- Semisimple B-field reductions $\Rightarrow$ unstable/stable extrema.
  - Extrema without a scaling nature.
  - The set of Mkw extrema draws closed lines in both the parameter space $(\theta_\epsilon, \theta_\xi)$ and the complex plane $\mathcal{Z}_0$.

Models based on the $\mathfrak{iso}(3)$ B-field reduction



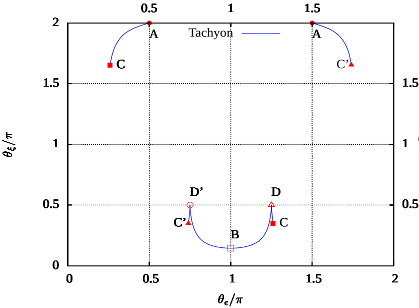
- Points A and A': Underlying $\mathfrak{g} = \mathfrak{iso}(3) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$, and $|\Phi_0| \rightarrow \infty$.
- Point B : Special point where $|\Phi_0| \rightarrow 0$.
- Line $\overline{\text{AA'}}$: Underlying $\mathfrak{g} = \mathfrak{so}(4) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$.

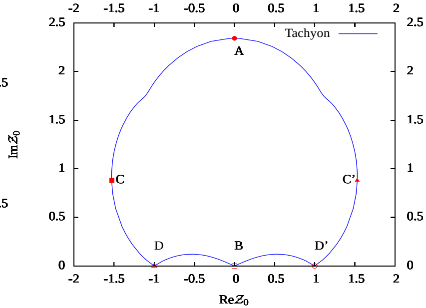
Models based on the $\mathfrak{su}(2)+\mathfrak{u}(1)^3$ B-field reduction



- Points A and A': Underlying $\mathfrak{g}=\mathfrak{iso}(3)\,+\,\mathfrak{nil}$, and $|\Phi_0|\rightarrow\infty$.
- Point B : Underlying $\mathfrak{g}=\mathfrak{so}(4)\,+\,\mathfrak{u}(1)^6$, and $|\Phi_0|\rightarrow 0$.
- Lines $\overline{\text{AB}}$ and $\overline{\text{BA'}}$: Underlying $\mathfrak{g}=\mathfrak{so}(4)\,+\,\mathfrak{nil}$.

Models based on the $\mathfrak{so}(4)$ B-field reduction



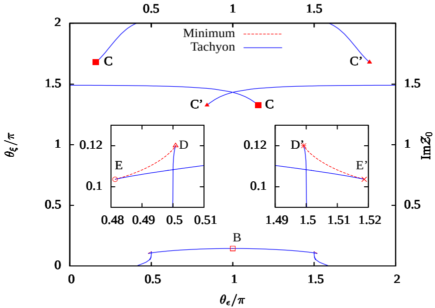


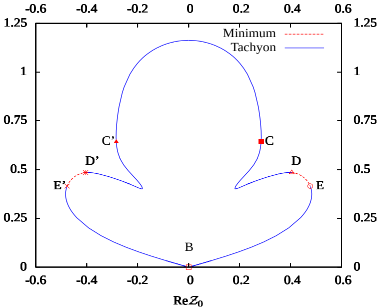
- Points D and D' : Underlying $\mathfrak{g} = \mathfrak{iso}(3) + \mathfrak{so}(4)$. At these points, $\text{Im}\mathcal{S}_0 \rightarrow \infty$ while $\text{Im}\mathcal{T}_0 \rightarrow 0$ and $\text{Im}\mathcal{Z}_0 \rightarrow 0$.

- Line $\overline{\text{DD'}}$ through point B: Underlying $\mathfrak{g} = \mathfrak{so}(4)^2$, and $\text{Im}\Phi_0 \rightarrow 0$ at point B.

- Line $\overline{\text{DD'}}$ through point A : Underlying $\mathfrak{g} = \mathfrak{so}(3,1) + \mathfrak{so}(4)$. Special A, C and C' points in which $\text{Im}\mathcal{T}_0 \rightarrow 0$ dynamically.

Models based on the $\mathfrak{so}(3,1)$ B-field reduction





- Unique $\mathfrak{g} = \mathfrak{so}(3,1)^2$ Supergravity algebra.
- Lines $\overline{\text{DE}}$ & $\overline{\text{D'E'}}$: Stable Mkw vacua!! $\Rightarrow$ They are continuously connected to dS stable vacua via the deformation $\theta_\xi \rightarrow \theta_\xi + \delta\theta_\xi$ with $0 < \delta\theta_\xi < \delta\theta_\xi^{(crit)}$.
- Point B: Special point where $\text{Im}\Phi_0 \rightarrow 0$.
- Points C and C' : Special points in which $\text{Im}\mathcal{S}_0 \rightarrow 0$ dynamically.

Review

- The set of Minkowski extrema can be obtained for the SUGRA models based on the five inequivalent B-field reduction.

- All these extrema become unstable except for a small region within the parameter space of those SUGRA models based on the $\mathfrak{so}(3,1)$ B-field reduction. This region has an underlying $\mathfrak{g} = \mathfrak{so}(3,1)^2$ Supergravity algebra and accommodates for non-supersymmetric stable Mkw vacua continuously connect to dS ones.

Type IIA dual Minkowski extrema

Objective

Understand the previous type IIB with O3/O7 orientifold models (and moduli extrema) from their type IIA with O6/D6 description.

Key points

- The mapping IIB $\leftrightarrow$ IIA between the contributions to the potential energy.
- No-go theorems concerning the existence of Mkw/dS extrema are formulated in a type IIA generalised flux language.

Type IIA with O6-planes and no-go theorems

- Fluxes and localised sources map between the IIB with O3/O7 and the IIA with O6 descriptions of the $\mathcal{N}=1$ type II orientifold models.

Description	IIB with O3/O7	IIA with O6
NS-NS fluxes	$\bar{H}_3\ ,\ Q$	$\bar{H}_3\ ,\ \omega\ ,\ Q\ ,\ R$
R-R fluxes	$\bar{F}_3$	$\bar{F}_0\ ,\ \bar{F}_2\ ,\ \bar{F}_4\ ,\ \bar{F}_6$
Sources type 1	O3/D3	O6/D6 (orient)
Sources type 2	O7/D7	O6/D6 (orient + orbif)

Shelton, Taylor and Wecht [arXiv:hep-th/0508133]
Aldazabal, Cámara, Font and Ibáñez [arXiv:hep-th/0602089]

- In the type IIA language, a few simple conditions for Mkw extrema to exist have been stated involving the flux-induced terms in V,

$$\begin{aligned} \left(V_{\omega} - V_{\bar{F}_2}\right) \quad + \quad 2\left(V_Q - V_{\bar{F}_4}\right) \quad + \quad 3\left(V_R - V_{\bar{F}_6}\right) \quad &= \quad 0 \\ \left(V_{\bar{F}_0} - V_{\bar{H}_3}\right) \quad + \quad \left(V_Q - V_{\bar{F}_4}\right) \quad + \quad 2\left(V_R - V_{\bar{F}_6}\right) \quad &= \quad 0 \end{aligned}$$

Hertzberg, Kachru, Taylor and Tegmark [arXiv:0711.2512 [hep-th]]
Haque, Shiu, Underwood and Van Riet [arXiv:0810.5328 [hep-th]]

From *non-geometric* IIB models to ...

- IIB with O3 models are non-geometric due to the Q flux $\Rightarrow$ IIA duals ?

- Computing the IIA dual flux-induced terms:

  - IIB models based on the B-field reductions of $\mathfrak{nil}$, $\mathfrak{iso}(3)$ and $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ (at the $\theta_\epsilon = \pm\frac{\pi}{2}$ circle), give rise to $V_Q = V_R = 0 \Longrightarrow$ *geometric* IIA models.

  - IIB models based on the B-field reductions of $\mathfrak{so}(4)$, $\mathfrak{so}(3,1)$ and $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ (far from the $\theta_\epsilon = \pm\frac{\pi}{2}$ circle), give rise to $V_Q \neq 0$ and/or $V_R \neq 0 \Longrightarrow$ *non-geometric* IIA models.

de Carlos, A.G, and Moreno [arXiv:0907.5580 [hep-th]]

Geometric type IIA Mkw extrema (Universal)

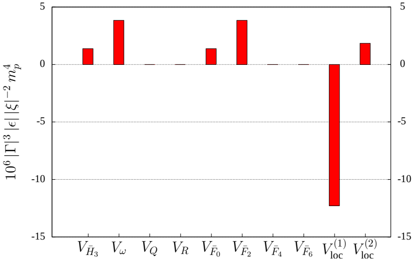
- Models based on $\mathfrak{g}_{gauge} = \mathfrak{iso}(3)$ with $\mathfrak{g} = \mathfrak{so}(4) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$.

- Unstable extrema.

- IIB models with a *geometric* IIA dual description:

$$V_\omega - \left(V_{\bar{F}_2} + 2V_{\bar{F}_4} + 3V_{\bar{F}_6}\right) = 0$$

$$V_{\bar{F}_0} - \left(V_{\bar{H}_3} + V_{\bar{F}_4} + 2V_{\bar{F}_6}\right) = 0$$



- Looking into the Re$\mathcal{S}$ and Re$\mathcal{T}$ axion stabilisation $\Rightarrow$ $V_{\bar{F}_4} = V_{\bar{F}_6} = 0 \Longrightarrow$ $V_{\bar{H}_3} = V_{\bar{F}_0}$ and $V_\omega = V_{\bar{F}_2}$ at any Mkw extremum.

- These Mkw extrema need type 1 O6-planes and type 2 D6-branes.

Non-geometric type IIA Mkw vacua

- Models based on $\mathfrak{g}_{gauge} = \mathfrak{so}(3,1)$ with $\mathfrak{g} = \mathfrak{so}(3,1)^2$.

- Region with stable vacua.

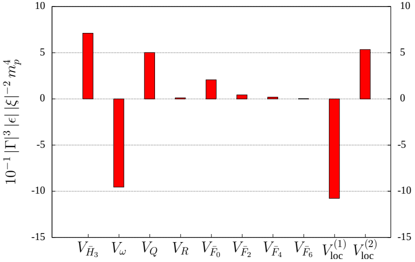
- Supersymmetry is broken:

$$F_{\mathcal{Z}} \neq 0 \ , \quad F_{\mathcal{S}} \neq 0 \ , \quad F_{\mathcal{T}} \neq 0$$

Gómez-Reino and Scrucca [arXiv:hep-th/0602246]

- F-term scalings:

$$F_{\mathcal{Z}} \propto |\xi| \quad \text{and} \quad F_{\mathcal{S}} \propto |\epsilon|$$



- dS saddle point with η-problem close to the Mkw vacuum.

Flauger, Paban, Robbins and Wrase [arXiv:hep-th/08123886]

Caviezel, Koerber, Körs, Lüst, Wrase and Zagermann [arXiv:hep-th/08123551]

- These Mkw vacua also require type 1 O6-planes and type 2 D6-branes.

Silverstein [arXiv:0712.1196 [hep-th]]

Conclusions

- A plethora of non-supersymmetric Mkw/dS classical extrema take place in $\mathcal{N} = 1$ type II orientifold models including generalised fluxes.

- Understanding the Supergravity algebras underlying these flux-induced models, becomes crucial for removing redundant degrees of freedom from the effective SUGRA models, and allows us to be exhaustive when performing a scanning of vacua.

- The Mkw extrema are found to describe lines in the parameter space connecting points associated to either a special algebra or a moduli space singularity.

- In the IIB with O3/O7 duality frame, non-supersymmetric Mkw/dS stable vacua exist for those SUGRA models based on $\mathfrak{g} = \mathfrak{so}(3,1)^2$, built from a $\mathfrak{g}_{gauge} = \mathfrak{so}(3,1)$ B-field reduction.

- Extended $\mathcal{N} \geq 2$ origin (if any) of these vacua based on gaugings at (e-m) angles?

de Roo, Westra and Panda [arXiv:hep-th/0606282]
A.G and Weatherill [arXiv:0811.2190 [hep-th]]
Aldazabal, Cámara and Rosabal [arXiv:0811.2900 [hep-th]]
Roest [arXiv:0902.0479 [hep-th]]

...thank you all !